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## COMMENTS ON THE TEACHING OF GEOMETRY.

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*Purpose or Problem.*—The writer of this paper has each day occasion to visit classes where geometry recitations are in progress in the course of his regular supervision work. Some outstanding features of the recitation work are here mentioned, that good classroom practices may become more common, and that practices of doubtful value may be called in question and, if found lacking, abandoned. Certain methods of work which seem effective in the study-recitation process are here considered in some detail and are recommended for trial and for adoption by those who succeed through their use.

*Recitation Routine.*—Within a given period a record was made of the work in progress at the time of the visit. This report shows that in a large majority of the cases some pupil was reading aloud the work of explanation which he had previously written in full at the board. In many cases the pupil, though standing, had maintained his position at or near his desk and was looking at and “talking at” a figure and written material some fifteen or twenty feet distant.

It is fair to assume that the teachers in charge must have given their approval to the method of explanation in progress. If so, then many teachers believe that good will somehow result from a re-reading of the material placed on the board. On the assumption that explanations are necessary and worth the time commonly devoted to them, it follows that such explanations should be given as effectively as possible.

Now effective presentation demands of the one who gives an explanation that he identify himself with the product offered. This identification of one's self with the product is made possible through taking a position near the diagram and by means of a pointer designating each line, point, angle, or surface referred to in the discussion. Some of the principles involved in good salesmanship should be practiced in the geometry classroom. Each pupil must be made to feel that he is selling his

method and work to his fellows. In the sale of his solution he should use such methods of appeal as will guarantee that each member of the class can see the part of the figure to which attention is directed. In practice, the figures used are often so small, and so located at the board as to be out of the range of vision of many members of the class. The figures used are commonly exact duplicates, letters and all, of those given in the text in use. No salesman would attempt to sell a shop-worn article. It must be new. The element of newness can be used if the figure used in the explanation presents the idea of the exercise in a different form or position. Then, too, a salesman shows but one article at a time and definitely directs the attention of the buyer to the essential qualities of the article he holds.

Again, it is relatively infrequent that one is privileged to observe a pupil take his place at the board and draw as he talks a large figure representing in an effective way each of the elements referred to in the theorem, and then to indicate his general plan of solution, saying, "I propose to prove that these two lines are equal by showing that they are corresponding parts of congruent triangles, or that they each equal a given line," etc. Instead, if the figure is not already at the board, in common practice the pupil silently draws the complete figure while the class awaits his pleasure. Such a practice teaches pupils to waste time. The real purpose of drawing the several lines is not evident to the class, and since no general plan of work has been announced, the completed diagram presents a needlessly complex situation to the pupils who really are in need of help.

How much better it would be were such construction lines as are used in explanations added in the development of the general plan of work which has been announced. The whole explanation would then be full of purpose and would be written more permanently into the nervous system of the pupil, for by original nature we are more responsive to such situations as involve movement than to those which are fixed and static. Since the plane of appeal involves motor neurone centers, the learning thus gained is more permanent.

*High-grade Service in Geometry Teaching.*—It is the con-

viction of the writer that the highest grade of service rendered by the geometry teacher is that given in problem situations where the pupil is making his first attack upon new theorems and exercises into which situations the teacher enters to guarantee that right habits of thought and work are learned and used. Good habits of thinking in geometry demand: (1) that the facts and relations of the given situation be clearly identified and represented; (2) that the direct outcomes of the given elements and required facts be noted; (3) that a search be made for such combinations of given elements as give promise of usefulness in reaching a situation in which the desired fact or relation will become evident; (4) that care be exercised in calling up and using theorems already proved and other known facts, to the end that the desired conclusion will be reached from known facts which have been properly related; and finally (5) that the materials of the proof be so arranged as to show clearly the correct reason for each step of the proof, with an evident logical arrangement of all materials used throughout the proof.

Where direction (teaching) of the type described above is not given when new theorems and problems are attacked, then the time of the teacher and class is largely consumed in rehearsing explanation given by pupils who have succeeded in their home work. Some teachers have discovered that a greater number of successes are experienced by pupils when at least half of the class period is spent with pupils as they make their first attack on new theorems and problems. Such teaching is vital, interesting to pupils, and stimulating to the teacher, for the needed help in developing and executing plans of attack can be given, and at the proper point.

In a very real sense pupils learn to do by doing under direction. They learn through succeeding in their work. If successes are experienced, learning will necessarily result. Many teachers find it necessary at the beginning of the pupil's work in geometry to guide pupils as they use each of the five steps listed in the previous paragraph. Later it is found necessary to direct the pupils through steps 1-4 only. Later still, through steps 1-3 only; and finally, when correct habits of thinking have been formed, little guidance need be given, ex-

cept in very difficult problem situations. There will always be need for guidance on the part of some pupils, though the need for this direct guidance should be reduced to the minimum.

Where the lesson assignment has been inadequate—that is, where certain of the steps listed above should have been taken under the direction of the teacher and were not taken—then a large portion of the time of the teacher and class must be consumed in the uninteresting routine of explaining solutions and correcting errors. Such explanations must be given and the time of the class must be thus utilized, unless *real teaching* is given, in making adequate lesson assignments and in directing the first work of pupils as they attack their new problem situations. Many teachers of geometry in Wisconsin have caught the vision of the better use of the class period. It has been the delight of the writer to visit a considerable number of classes where *geometry is really taught*.

In good teaching of geometry motivated drills must be given and difficulties must be cleared up for those who have such. Certain problems must from time to time be worked out in full in lesson assignments to guarantee that correct habits of thought and of work are being utilized. An inventory of the advance lesson must be taken.

In preparing to assign the lesson, the teacher will either take sufficient time to work through the several exercises to be assigned or he will have made use of such teaching helps as are provided. It is the conviction of the writer that the study necessary for the teaching of a lesson should be given in preparation for the assignment of that lesson. Only under such conditions can adequate lesson assignments be made.

The following rule may safely be followed: Give as much weight in thought, in time, and in practice to pupils as they attack new situations as you do in rehearsing the details of old situations. Where such a division of the class period is made, due consideration will be given to the teaching of pupils in methods of attack upon new situations and in forming the most profitable habits of thought and of work. In short, pupils will be taught how to think through the guidance given by the teacher in problem situations which require real thinking.

Then, too, sufficient time will be available for checking up the work of pupils and for the giving of help to those who need it.

*Lesson Assignments Based on a Recognition of Individual Differences.*—Teachers agree in theory at least that each pupil should be given enough to do to tax his total mental capacity to the full for the time allotted to the subject for class work and home study. Knowing as we do that some pupils can do several times as many exercises as others in a given time, the ideal would be reached only through individual instruction where each pupil progresses at his own maximum rate. Our plan for class instruction makes such a provision for lesson assignments well nigh impossible.

Again, we might conceive of our classes as sectioned into given groups on the score of ability so as to place in the failure, inferior, medium, superior and excellent groups, 5 per cent., 20 per cent., 50 per cent., 20 per cent. and 5 per cent, respectively, and then to plan our lesson assignments so as to adapt them to the needs of each of the five groups. Such a plan, too, is doubtless impracticable for the small and median high school, at least for the present time. We can, however, and I think we should, so plan the extent of the lesson assignments as to lay out a certain amount of work, theorems and exercises, for the lower 75 per cent. of the class which we may designate as *required work*, and offer in addition a certain amount of work (number of exercises) for the superior and excellent pupils, which work we, in lesson assignments, might designate as *optional*. The place of the pupil in the upper 25 per cent. of the class clearly depends upon his willingness to do and his success in doing the optional work.

The emphasis given previously in this article on lesson assignments relates, of course, to the work here designated as required. Little direction, if any, should be given to the optional work as far as lesson assignment is concerned. This optional work should be voluntary and largely self-directed. Reports on this optional work when made to the class afford much interest and inspiration to the entire class.

*Reviews and Drills.*—Good teaching demands that such subject matter as is worth remembering be called up before it has dropped below the level of consciousness. Psychologists show

us that the most rapid drop in the curve of forgetting occurs in the intervals which elapse immediately following the learning of the content. Since the evidence is available and reliable, it follows that good teaching should include provision for effective drills and reviews. Such drills should be quickly carried out and frequently given.

In a few cases the writer has found that the figures for the theorems have been drawn with ink on heavy 24 x 36 inch paper. These figures are drawn carefully with black waterproof ink and differ materially, though in no essential particulars, from the figures given for the theorems in the text in use. The figures are, of course, lettered differently and are filed away in the teacher's cabinet, where they are ready for almost instant use. The use of these figures is recommended for reviews only. Their use will more than double the amount of material or topics which can be reviewed in a given period.

*Thinking with Pupils on Original Work.*—The writer recommends that from time to time teachers of geometry select original exercises with the solution of which they are not familiar, and that they set for themselves the tasks of writing out the solutions of the problems, making a written record of each idea, hint, or suggestion that is hit upon in the course of the solution. Such a record would in some cases be a striking revelation of the method of trial and success as the only method available in new problem-solving situations. These solutions have been found particularly interesting and helpful when carried out in line with the five steps previously mentioned in this article. In any case, the teacher who takes the trouble to make such a written record of the workings of his own mind in new problem situations will better appreciate the difficulties with which pupils are constantly surrounded.

Teachers find that the written record referred to above serves as a splendid guide in teaching pupils how to succeed in solving original exercises of sufficient difficulty to tax to the full their total mental equipments. The solving of originals by the teacher, both at home and in the classroom, gives the only adequate preparation for the teaching of pupils in the methods of attack and solution of new problem-solving situations.